

4/9/18 "The strength to win comes from within."-Anonymous

HW: "Horizontal Stretching of functions" homework section

Quarter Test Thursday 4/12

AIM: How do we recognize a horizontal stretch?

Warm Up:

The function $h(x)$ has a range given by the interval $[-2, 8]$. The function $f(x)$ is defined by $f(x) = \frac{1}{2}h(x) + 6$.

What is the range of $f(x)$?

$$-2 \leq y \leq 8$$

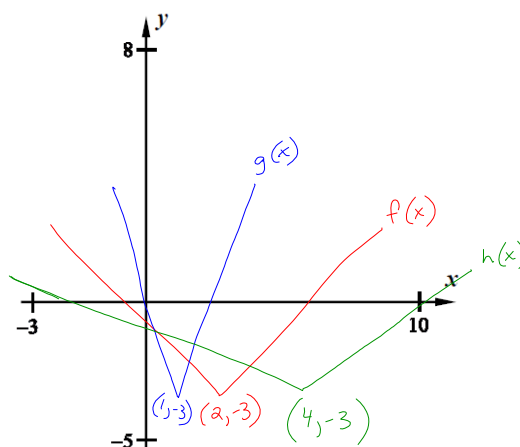
$$\begin{aligned} -2 \times \frac{1}{2} &= -1 + 6 = 5 \\ 8 \times \frac{1}{2} &= 4 + 6 = 10 \end{aligned}$$

$$[5, 10]$$

$|x|$ right 2 down 3

Exercise #1: Consider the absolute value function $f(x) = |x - 2| - 3$.

- (a) Using your calculator, sketch a graph of f on the axes provided. Label the coordinates of its vertex point without the use of your calculator.



- (b) Consider the function $g(x) = f(2x)$. Determine a formula for g and then graph it on the axes. Use your calculator to find its minimum point and label it on the graph.

$$f(x) = |x - 2| - 3$$

$$f(2x) = |2x - 2| - 3$$

$$g(x) = |2x - 2| - 3$$

Minimum
point
(1, -3)

- (c) Now consider the function $h(x) = f\left(\frac{1}{2}x\right)$. Determine a

formula for h and graph it on the axes. Use your calculator to find its minimum point and label it on the graph.

$$f(x) = |x - 2| - 3$$

$$f\left(\frac{1}{2}x\right) = \left|\frac{1}{2}x - 2\right| - 3$$

$$h(x) = \left|\frac{1}{2}x - 2\right| - 3$$

- (d) Summarize your findings below for each function.

$f(x)$ turning point:

(2, -3)

$f(2x)$ turning point:

(1, -3)

$f\left(\frac{x}{2}\right)$

$f\left(\frac{1}{2}x\right)$ turning point:

(4, -3)

- (e) What stayed constant about the turning points? What changed and how did it change?

The y-value
stayed

The x-values
changed by doing the
inverse (opposite) of what
it showed

HORIZONTAL DILATIONS

For a real number, positive constant such that $k > 1$:

1. The function $f(kx)$ represents a horizontal compression of $f(x)$ by a factor of k
shrink
2. The function $f\left(\frac{1}{k}x\right)$ represents a horizontal stretch of $f(x)$ by a factor of k .

Exercise #2: Let's take a look at the quadratic function $f(x) = x^2 - 12x + 20$.

- (a) Determine the coordinates of its turning point by using the equation for the axis of symmetry of

$$x = -\frac{b}{2a}$$

$$x = \frac{-(-12)}{2(1)} = \frac{12}{2} = 6$$

$$a = 1$$

$$b = -12$$

$$c = 20$$

$$f(6) = 6^2 - 12(6) + 20$$

$$= 36 - 72 + 20$$

$$f(6) = -16$$

$$T.P. = (6, -16)$$

- (b) If g is defined by $g(x) = f(3x)$, what should be the coordinates of its turning point based on our previous work? Explain.

divide x-value by 3

$$\text{New T.P.} = (2, -16)$$

- (c) Determine a formula for $g(x)$ and then use the turning point formula to verify your answer from part (b).

$$g(x) = f(3x)$$

$$g(x) = f(3x) = (3x)^2 - 12(3x) + 20$$

$$g(x) = 9x^2 - 36x + 20$$

$$x = -\frac{b}{2a} = \frac{-(-36)}{2(9)} = \frac{36}{18} = 2$$

$$g(2) = 9(2)^2 - 36(2) + 20$$

$$= 36 - 72 + 20$$

$$g(2) = -16$$

$$T.P. = (2, -16)$$

- (d) Show that the y-intercept of both $f(x)$ and $g(x)$ are equal. What does this make sense from a horizontal dilation perspective?

@ y-intercept $x = 0$

$$f(0) = 0^2 - 12(0) + 20 = 20$$

$$g(0) = 9(0)^2 - 36(0) + 20 = 0 - 0 + 20 = 20$$

Transformations

Inside

x-values
change

$f(x+2)$ left 2

$f(x-3)$ right 3

$f(-x)$ reflect over
y-axis

$f(3x)$ shrink by
a factor of 3

$f(\frac{1}{2}x)$ stretch by
a factor of 2

⊗ We do the
inverse (opposite)
of what is shown
to the x-values

Outside

y-values change

$f(x)+2$ up 2

$f(x)-4$ down 4

$-f(x)$ reflect over x-axis

$3f(x)$ stretch by a factor
of 3

$\frac{1}{2}f(x)$ shrink by a factor
of $\frac{1}{2}$

⊗ Do exactly what
is shown to the
y-values